# 1 Last Lecture

Magnetic force on a wire independent of shape;

Motion of a charged particle in uniform $\vec{B}$;

Cyclotron radius: $r = \frac{mv}{qB}$ meters

Cyclotron frequency: $\omega = \frac{qB}{m}$ radian Hertz.

# 2 Today

C Charged particle motion in magnetic field (last lecture)

   2 Lorentz force law

D Torque on current loop – magnetic dipole moment

   1 Torque & magnetic moment

   2 Potential energy

   3 Gyromagnetic M/L

E Hall Effect

# 3 And the actual notes . . .

Professor McKee was away doing something awesome with a massive telescope probe, which will be in the shadow of the earth to look at the infrared spectrum. They want to find the light from the first stars in the universe. Will be launched in June 2015.

NB that $\frac{mv}{qB} = \frac{p}{qB}$. Neat.

## C Charged particle in a magnetic field (last lecture)

### 2 Lorentz force law

$\vec{F}_e = q\vec{E}$, $\vec{F}_b = q(\vec{v} \times \vec{B})$. Total electromagnetic force on a charged particle $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$. This holds true even when dealing with velocities close to $c$.

Let's work out an example: what if we put a charged particle in crossed $\vec{E}$ and $\vec{B}$ fields, i.e., $\vec{E}$ perpendicular to $\vec{B}$. If the particle moves at a velocity $\vec{v} = \frac{\vec{E} \times \vec{B}}{B^2}$ $(E << B)$, then $\vec{F} = 0$. That is to say, there's a specific velocity that the particle can move with without any force exerted.

Remember that $\vec{A} \times (\vec{B} \times \mathcal{C}) = \vec{B}(\vec{A} \cdot \mathcal{C}) - \mathcal{C}(\vec{A} \cdot \vec{B})$; then

$$\vec{F}_B q^{-1} = \vec{v} \times \vec{B} = \frac{(\vec{E} \times \vec{B}) \times \vec{B}}{B^2}$$

$$= (\vec{E} \times \hat{B}) \times \hat{B} = -\hat{B} \times (\vec{E} \times \hat{B})$$

$$= -[\vec{E}(\hat{B} \cdot \hat{B}) - \hat{B}(\hat{B} \cdot \vec{E})] = -\vec{E}(\Leftarrow \hat{B} \perp \vec{E})$$

## D Torque on Current Loop

### 1 Magnetic Dipole Moment

Recall that $\vec{p} = q\vec{d}, \vec{\tau} = \vec{p} \times \vec{E}, U = -\vec{p} \cdot \vec{E}$, we want something similar for a magnetic dipole.

Consider a planar current loop with top length $b$ and side length $a$ with a current $I$ running right-to-left; an area vector points normal to the plane at some angle $\theta$ to the vertical; $\vec{B}$ is going "up".

First, we know that $\vec{F} = I\vec{\ell} \times \vec{B}$; forces on the opposite sides cancel, since $\vec{\ell} \parallel I$. But what's the torque? $\vec{\tau} = \vec{r} \times \vec{F}$; the roll torque ($b$-length) of the current loop is 0: $\vec{\tau} = \vec{r} \times \vec{F} = \frac{\vec{b}}{2} \times \vec{F} = 0$, since $\vec{b} \parallel \vec{F}$.

How about the pitch? $\tau = \frac{a}{2} IbB \sin\theta$ on each; so $\sum \tau = abIb \sin\theta$. Since $A = ab$, $\tau = IAB \sin\theta$, with vectors, have that $\vec{\tau} = I\vec{A} \times \vec{B}$.

If we have $n$ loops of wire, the total current is $nI$, so the torque becomes $nI\vec{A} \times \vec{B}$.

Then define the magnetic dipole moment, $\vec{\mu} \equiv nI\vec{A}$, so $\vec{\tau} = \vec{\mu} \times \vec{B}$.

## 2 Potential energy

First, let's record our expectations – given the diagram as before, the torque will align $\vec{\mu} \uparrow\uparrow \vec{B}$.

Work done by field, $dW = -\tau d\theta$; $dU = -dW = \tau d\theta = \mu B \sin\theta d\theta \Rightarrow U = -\mu B \cos\theta + c$, choose the constant so that $U = 0 \longleftrightarrow \theta = \frac{\pi}{2}$, so $U = -\vec{\mu} \cdot \vec{B}$, implying that the state of maximum energy is when $\vec{\mu} \uparrow\downarrow \vec{B}$.

## 3 Gyromagnetic ratio

Gyromagnetic ratio is the ratio of the magnetic dipole moment to the angular momentum. They seem totally unrelated, but we'll see that it actually just comes down to the ratio of the charge and the mass.

Take, e.g., a point charge orbiting in uniform $B$. $\mu = IA$, $I = \frac{q}{T}$ $(vT = 2\pi r)$. $\Rightarrow I = \frac{qv}{2\pi r}$, so $\mu = IA = \frac{qv}{2\pi r}\pi r^2 = \frac{1}{2}qvr$.

Now, what's the angular momentum? $L = mrv \rightarrow \frac{\mu}{L} = \frac{1}{2}\frac{qvr}{mrv} = \frac{1}{2}fracqm$, which is one-half the charge-to-mass ratio.

This has a direct application to magnetic materials: in the Bohr model of the atom, we have a proton at the center with an electron orbiting it; the essence of Bohr's model is that the angular momentum is quantized. $L = \hbar = \frac{h}{2\pi}$; $\mu = \frac{1}{2}\frac{e}{m_e}\hbar$. If we put in numbers, $\mu = 9.27 \times 10^{-24}$ J T$^{-1}$, a typical value for an atomic mgnetic dipole.

# E Hall Effect

Named for the physicist who discovered it in the later part of the 19th century.

Suppose a wire with a current $I$ flowing to the right in a magnetic field flowing away from us; therefore the force $I\vec{\ell} \times \vec{B}$ is pointing upwards.

$\vec{F} = q(\vec{E}_{current} + \vec{v}_d \times \vec{B} + \vec{E}_H)$, require that $\vec{F} = q(\vec{E}_{current} \rightarrow q\vec{E}_H = -q\vec{v}_d \times \vec{B}$.

If wire has diameter $d$, Hall emf $= \xi = E_H d = v_e B d$.